Exam Advanced Logic

Instructions:

- Put you name and student number on the top left of the first page.
- Put your name on subsequent pages as well.
- Do not use pencil or a red pen to make your exam.
- Leave ten lines blank below your name on the first page.
- Motivate all your answers.
- If you want to inspect your exam after it has been graded, you can do so by making an appointment with Barteld Kooi, tel. 050-3636924.
- Please fill in the anonymous course evaluation

Good luck!

1. **Induction (10 pt)** Consider the language of propositional logic without negation, i.e. the *neg-free-wffs*.

   i Each propositional letter p is a neg-free-wff.

   ii If A and B are neg-free-wffs, then so are $(A \wedge B)$, $(A \vee B)$, $(A \rightarrow B)$ and $(A \leftrightarrow B)$.

   iii Nothing is a neg-free-wff unless it is generated buy repeated applications of i and ii.

   Also consider the valuation v that assigns truth value 1 to all propositional letters.

   Show by induction that $v(A) = 1$ for all neg-free-wffs.

2. **Three-valued logics (10 pt)** Determine whether the following holds in RM_3 using a truth table.

$$p \supset (q \wedge r) \models (r \supset \neg q) \wedge (\neg r \supset p) \wedge p$$

3. **FDE tableau (10 pt)** By constructing a suitable tableau determine whether the following formula is valid in **FDE**. If the formula is invalid, provide a counter model.

$$(\neg(\neg p \vee q) \vee p) \supset p$$

4. **Fuzzy logic (10 pt)** Determine whether the following holds in $Ł_{\aleph}$ (where $D = \{1\}$). If so, show that if the premises have value 1, so does the conclusion. If not, provide a counter-model.

$$\models ((p \vee q) \wedge (p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow r$$

5. **Basic modal tableau (10 pt)** By constructing a suitable tableau determine whether the following formula is valid in K. If the formula is invalid, provide a counter model.

$$\Box p \supset \Box(\Box p \supset \Box\Box p)$$

6. **Normal modal tableau (10 pt)** By constructing a suitable tableau determine whether the following formula is valid in $K\rho$. If the formula is invalid, provide a counter model.

$$p \supset (\Box(p \supset \Box p) \supset \Box p)$$

7. **First-order modal tableau (10 pt)** By constructing a suitable tableau determine whether the following formula is valid in $VK^t\eta$. If the formula is invalid, provide a counter model.

$$\langle P\rangle[F](\exists x Px \supset [F]\exists x Px) \supset (\exists x Px \supset \neg\langle F\rangle\forall x\ \neg Px)$$

8. **Soundness and completeness (10pt)** Consider the following tableau in K, which contains only one branch which we call b

$$\begin{array}{c} \neg(\Diamond\Diamond p \supset \Diamond p), 0 \\ \Diamond\Diamond p, 0 \\ \neg\Diamond p, 0 \\ \Box\neg p, 1 \\ 0r1 \\ \Diamond p, 1 \\ \neg p, 1 \\ 1r2 \\ p, 2 \end{array}$$

   (a) Is b a *complete* branch?
   (b) Branch b is *open*. Provide an interpretation $\mathcal{I}$ and a function f such that f shows that $\mathcal{I}$ is *faithful* to b.

9. **Default logic (10 pt)** Consider the folowing set of default rules:

$$D = \left\{ \delta_1 = \frac{S(x) : Q(x) \wedge R(x)}{T(x)}, \qquad \delta_2 = \frac{T(x) : \neg R(x)}{P(x)}, \right.$$

$$\left. \delta_3 = \frac{S(x) : T(x)}{P(x)}, \qquad \delta_4 = \frac{P(x) \vee T(x) : S(x)}{R(x)} \right\},$$

and initial set of facts:

$$W = \{\forall x P(x) \to \neg Q(x), S(e)\}.$$

You only need to apply the default rules to the relevant constant e. Recall that a formula φ is a *sceptic consequence* if and only if φ is true in every extension of (W, D), while it is a *credulous consequence (goedgelovig gevolg)* if and only if φ is true in some extension of (W, D).

   (a) Draw the process tree of this default theory.
   (b) Is $T(e)$ a sceptic consequence of this theory?
   (c) Is $P(e)$ a credulous consequence of this theory?